# WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

*Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.*

| Part | What it is | How to use it | Time |
|---|---|---|---|
| 1 | Learn First | Read and annotate. Copy every worked example by hand. | 25 min |
| 2 | Worked Examples | Cover the solution. Try. Then compare line by line. | 20 min |
| 3 | Guided Practice | Steps are given; you fill the gaps. | 15 min |
| 4 | Practice Set A · Fluency | Straight application. Aim for speed and accuracy. | 20 min |
| 5 | Practice Set B · Reasoning | Multi-step. Show every line. | 25 min |
| 6 | Practice Set C · Challenge | Stretch problems. One may beat you — that is fine. | 25 min |
| 7 | The Crossnumber | Answers are numbers. Wrong maths will not fit. | 20 min |
| 8 | Number Search & Magic Square | Solo or pairs. | 15 min |
| 9 | Spot the Mistake | Audit a worked solution. Find the planted error. | 15 min |
| 10 | Quick-Fire Round | Matching, True/False, Two Truths and a Lie. | 15 min |
| 11 | The Escape Challenge | Teams of 4. | 25 min |

# Part 1 · Learn First

*A circle looks simple — round, symmetric, the same all the way round. But that one property (every point the same distance from the centre) is enough to prove a whole chain of surprising facts about chords, angles and quadrilaterals. This chapter is about proof, not calculation — read every "why is this true?" as carefully as the statement itself.*

## What a circle actually is

Before anything else, pin down the definition precisely — every theorem in this chapter comes back to it.

**Definition.** A circle is the set of all points on a plane that are equidistant from a given point on that plane. That fixed point is the **centre**; the common distance is the **radius**.

A set of points satisfying a condition is called the **locus** of points satisfying that condition — so a circle is the locus of points at a fixed distance from a given point.

If B and C are two points on the circle, the segment BC is a **chord**. A chord passing through the centre is a **diameter** — the longest possible chord. The angle BAC (A the centre) is the angle the chord **subtends** at the centre.

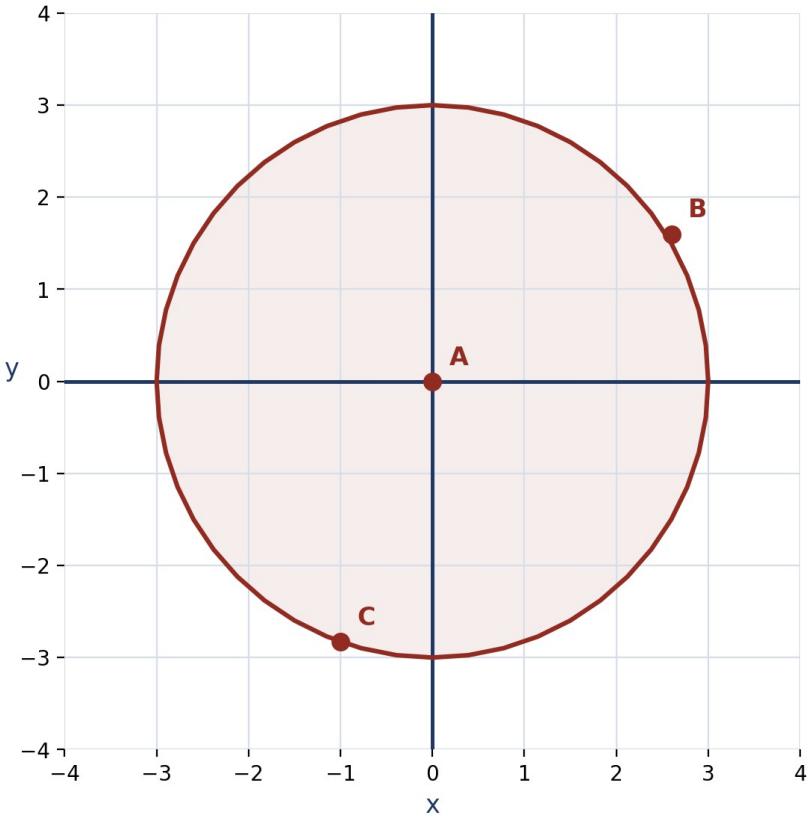


*Fig. 5.1 — Circle with centre A and chord BC*

### Rule to remember

- Every diameter is a line of **reflection symmetry** of the circle.
- A circle has complete **rotational symmetry** — rotate it by any angle

about the centre and it looks unchanged.

- The diameter is the longest chord; its distance from the centre is 0.

## How many circles pass through given points?

Through two points, infinitely many circles pass. Through three non-collinear points, there is exactly one.

Any point equidistant from A and B is a valid centre for a circle through A and B. The **locus** of such points is the perpendicular bisector of AB. So every point on that perpendicular bisector is the centre of some circle through A and B — infinitely many circles.

**Theorem 1.** There is a unique circle passing through three non-collinear points A, B, C. Its centre O satisfies OA = OB (so O lies on the perpendicular bisector of AB) and OA = OC (so O lies on the perpendicular bisector of AC too). Two non-parallel lines meet at exactly one point, so O is fixed — and the circle of radius OA centred at O passes through all three.

This O is the **circumcentre** of triangle ABC; the circle is the **circumcircle**. If A, B, C are collinear, no circle passes through all three (the "perpendicular bisectors" would have to be parallel, and never meet).

### Where the circumcentre sits

- **Acute-angled** triangle — circumcentre lies **inside** the triangle.
- **Obtuse-angled** triangle — circumcentre lies **outside** the triangle.
- **Right-angled** triangle — circumcentre lies at the **midpoint of the hypotenuse**.

## Chords and the angles they subtend

Equal chords and equal central angles come as a pair — each implies the other.

**Theorem 2.** Equal chords of a circle subtend equal angles at the centre. If AB = DE, then in triangles CAB and CDE we have CA = CB = CD = CE = r (all radii) and AB = DE (given), so by **SSS** the triangles are congruent and ∠ACB = ∠DCE.

**Theorem 3 (converse).** Chords that subtend equal angles at the centre are equal in length. Here CA = CB = CD = CE = r and ∠ACB = ∠DCE (given), so by **SAS**, △ACB ≅ △DCE, giving AB = DE.

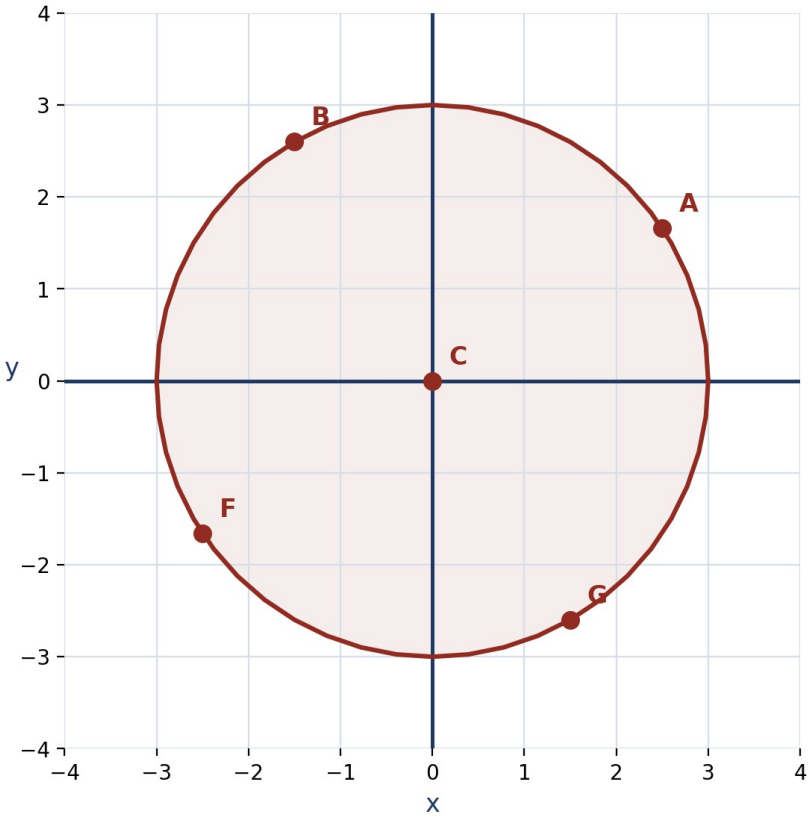


*Fig. 5.2 — equal chords AB and FG are equidistant from centre C*

## Midpoints, perpendicular bisectors, and distance from the centre

The centre, the midpoint of a chord, and the perpendicular to that chord are always tangled together.

**Theorem 4.** The line joining the centre to the midpoint M of a chord AB is perpendicular to AB. Proof sketch: △CAB is isosceles (CA = CB = r), so ∠A = ∠B; with AM = BM, △CMA ≅ △CMB by SAS, so ∠CMA = ∠CMB, and since these two angles form a straight line they must each be 90°.

**Theorem 5 (converse).** The perpendicular from the centre to a chord bisects the chord.

**Theorem 6.** Chords of equal length are equidistant from the centre. **Theorem 7 (converse).** Chords equidistant from the centre are equal in length.

**Theorem 8.** Of two unequal chords, the **longer** chord is **closer** to the centre. Using the Baudhāyana–Pythagoras theorem on the two right triangles formed by the perpendiculars, a bigger chord forces a smaller perpendicular distance.

$$chord\,length = 2\sqrt{r^2 - d^2}$$

### Chord-distance formula

- If a chord is at perpendicular distance $d$ from the centre of a circle of radius $r$, its length is $2\sqrt{r^2 - d^2}$.
- This comes straight from the Baudhāyana–Pythagoras theorem applied to the right triangle formed by the radius, the half-chord, and the perpendicular.

## Angles subtended by an arc

An arc subtends an angle at the centre, and a *different* (but related) angle at any point on the rest of the circle.

An **arc** is a connected piece of the circle between two end points. The smaller piece is the **minor arc**; the larger is the **major arc**. The angle subtended by the arc at the centre is measured by sweeping from one radius to the other **along that arc**.

**Theorem 9.** The angle subtended by an arc at the centre is **double** the angle it subtends at any point on the circle outside the arc. This uses the exterior-angle theorem on the isosceles triangles formed by the radii.

**Corollary.** The angle subtended by a diameter at any point on the circle is 90° — the arc from A to B not containing D subtends a straight angle (180°) at the centre, so it subtends half of that, 90°, at D.

**Angles in the same segment are equal.** For any two points D, E on the circle, outside arc AB and on the same side of it, ∠ADB = ∠AEB, since both equal half the central angle of arc AB.

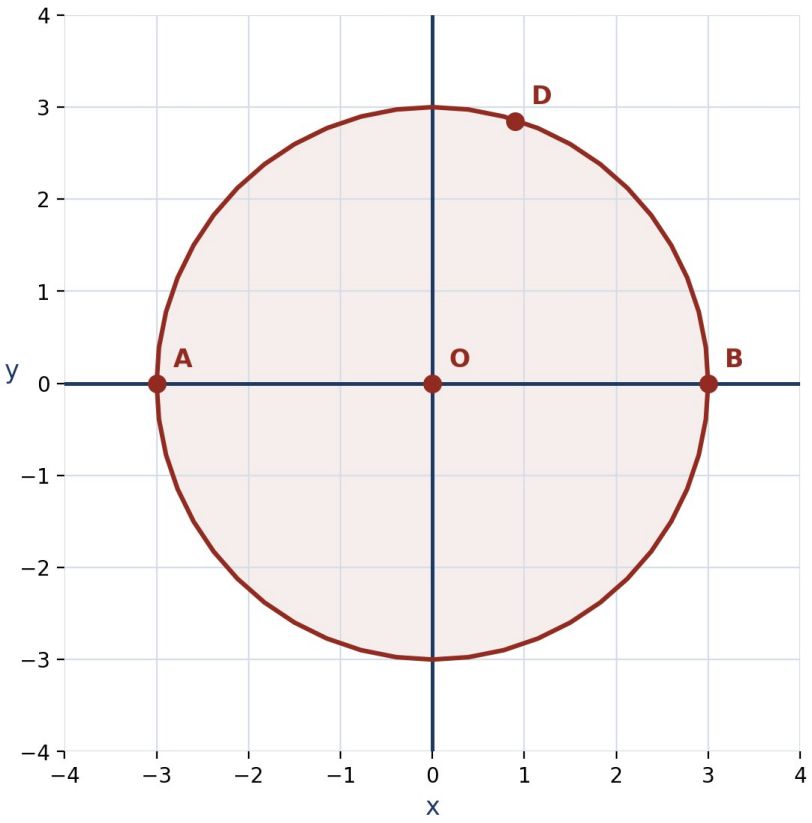


*Fig. 5.3 — the angle in a semicircle, ∠ADB, is always 90°*

## Concyclic points and cyclic quadrilaterals

When do four points lie on one circle? And what does that force about the quadrilateral they form?

Points on the same circle are called **concyclic**.

**Theorem 10.** If segment AB subtends equal angles at two points C, D on the same side of AB, then A, B, C, D are concyclic. (Proved by contradiction: if D were not on the circle through A, B, C, an exterior-angle argument forces an angle to be strictly greater than itself.)

A quadrilateral whose vertices are concyclic is a **cyclic quadrilateral**.

**Theorem 11.** The sum of a pair of opposite angles of a cyclic quadrilateral is 180°. Each angle is half of the central angle subtended by the opposite arc, and the two arcs together sweep the full 360° at the centre, so the two angles add to half of 360°, i.e. 180°.

**Theorem 12 (converse).** If a pair of opposite angles of a quadrilateral sum to 180°, the quadrilateral is cyclic.

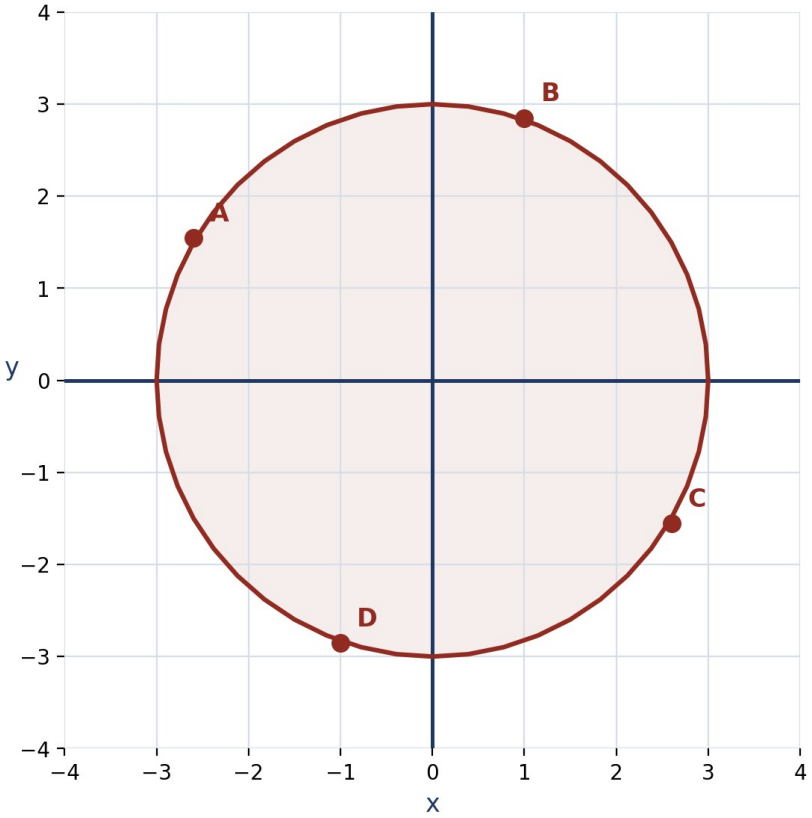


*Fig. 5.4 — cyclic quadrilateral ABCD; ∠A + ∠C = 180°*

### Rule to remember

- Cyclic quadrilateral: **opposite angles sum to 180°** — both ways, as a test and as a consequence.
- Exterior angle of a cyclic quadrilateral = the **interior opposite angle**.

# Part 2 · Worked Examples

*Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.*

## Example 1 — A chord is 5 cm from the centre of a circle of radius 13 cm. Find its length

**Step 1.** Use the chord–distance formula: length ¿$2\sqrt{r^2-d^2}$.

**Step 2.** Here $r=13$, $d=5$, so $r^2-d^2=169-25=144$.

**Step 3.** $\sqrt{144}=12$.

**Answer:** length ¿$2\times12=24$ cm.

> **Why this works**
>
> This is really the Baudhāyana–Pythagoras theorem in disguise: radius, half-chord and perpendicular distance form a right triangle with the radius as hypotenuse.

## Example 2 — An arc subtends 80° at the centre. Find the angle it subtends at a point on the major arc

**Step 1.** By Theorem 9, the central angle is double the angle at any point on the circle outside the arc.

**Step 2.** So the angle at the point equals half the central angle.

**Step 3.** $\frac{80^\circ}{2}=40^\circ$.

**Answer:** 40°.

> **Why this works**
>
> Every point on the major arc, no matter where, gives exactly the same 40° — that is the content of "angles in the same segment are equal".

## Example 3 — ABCD is a cyclic quadrilateral with ∠A = 75°. Find ∠C

**Step 1.** In a cyclic quadrilateral, opposite angles sum to 180° (Theorem 11).

**Step 2.** ∠A and ∠C are opposite, so ∠A + ∠C = 180°.

**Step 3.** ∠C = 180° − 75° = 105°.

**Answer:** ∠C = 105°.

> **Why this works**
>
> Opposite means across the quadrilateral, not adjacent — A pairs with C, and

> B pairs with D.

# Part 3 · Guided Practice

*The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.*

**1.** A chord of a circle of radius 10 cm is at a distance of 6 cm from the centre. Find its length.

Use length ¿ $2\sqrt{r^2-d^2}$ with $r=$¿ _____ and $d=$¿ _____ .

$r^2-d^2=100-$¿¿

$\sqrt{¿}=¿¿¿$, so length ¿ $2\times$¿ =¿ _____ cm.

---

**2.** An arc subtends 130° at the centre of a circle. What angle does it subtend at a point on the circle outside the arc?

By Theorem 9, the central angle is _____ times the angle at the circle.

So the angle at the circle is $\frac{¿^\circ}{2}=$¿ _____ .

---

**3.** PQRS is a cyclic quadrilateral with ∠P = (2x+10)° and ∠R = (3x−20)°. Find x.

∠P and ∠R are opposite angles, so ∠P + ∠R = _____ °.

$(2x+10)+(3x-20)=180$, giving $5x-10=$¿ _____ .

$5x=$¿ _____ , so $x=$¿ _____ .

---

**4.** AB is a diameter of a circle and C is any point on the circle. What is ∠ACB, and why?

AB is a diameter, so the arc not containing C subtends a _____ ° angle (a straight angle) at the centre.

By the Corollary to Theorem 9, ∠ACB is _____ that central angle.

So ∠ACB = _____ °.

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# Practice Set A · Fluency

*Straight application of the method. Aim to get every one right, then aim to get them right quickly.*

| # | Question | Your working and answer |
|---|---|---|
| 1 | A chord of a circle is 24 cm long and is 5 cm from the centre. Find the radius. | |
| 2 | The radius of a circle is 17 cm and a chord is 8 cm from the centre. Find the length of the chord. | |
| 3 | An arc subtends 96° at the centre. What angle does it subtend at a point on the circle outside the arc? | |
| 4 | AB is a diameter of a circle and C lies on the circle. If ∠BAC = 35°, find ∠ACB and ∠ABC. | |
| 5 | ABCD is a cyclic quadrilateral. If ∠A = 68° and ∠B = 112°, find ∠C and ∠D. | |
| 6 | Two chords of a circle are equal in length. What can you say about their distances from the centre? | |
| 7 | Of two chords AB and CD in the same circle, AB is longer than CD. Which is closer to the centre? | |
| 8 | State, in your own words, the definition of a circle as a locus. | |
| 9 | A circle passes through three non-collinear points. What is the special name for its centre? | |
| 10 | PQRS is a cyclic quadrilateral. If ∠P = 95°, find ∠R. | |

# Practice Set B · Reasoning

*Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.*

| # | Question | Your working and answer |
|---|---|---|
| 1 | A chord of length 16 cm is drawn in a circle of radius 10 cm. Find its distance from the centre. | |
| 2 | Two parallel chords of lengths 6 cm and 8 cm lie on opposite sides of the centre of a circle of radius 5 cm. Find the distance between the chords. | |
| 3 | In a circle with centre O, the central angle AOB = 60° and the radius is 12 cm. Find the length of chord AB. (Hint: triangle OAB is equilateral.) | |
| 4 | ABCD is a cyclic quadrilateral. E is a point on the extension of CD beyond D. Explain why ∠ADE = ∠ABC. | |
| 5 | Two parallel chords of lengths 10 cm and 24 cm lie on the same side of the centre of a circle, 7 cm apart. Find the radius of the circle. | |
| 6 | A cyclic quadrilateral has angles 80°, 110°, 100°, 70° in order. Is this possible? Explain. | |
| 7 | If the distance of a chord AB from the centre is twice the distance of chord CD from the centre, is CD = 2·AB? Explain why or why not. | |
| 8 | MNOP is a cyclic quadrilateral in which MN is a diameter. What can you say about ∠MOP and ∠MNP? | |

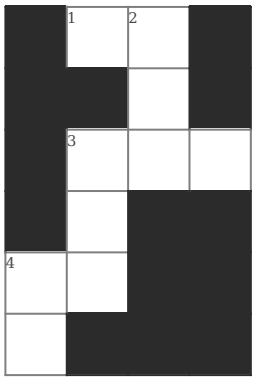
# Practice Set C · Challenge

*Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.*

| # | Question | Your working and answer |
|---|---|---|
| 1 | Prove that the perpendicular bisector of any chord of a circle passes through the centre. | |
| 2 | Two chords AB and CD of a circle intersect. Show that if AB = CD, the segments of one chord equal the corresponding segments of the other. | |
| 3 | A regular hexagon is inscribed in a circle of radius r. Find the side length of the hexagon and the distance of each side from the centre. | |
| 4 | Show that a rectangle is the only kind of parallelogram that can be inscribed in a circle. | |
| 5 | Let A be a point inside a circle with centre O (not the centre itself). Show that the shortest chord through A is the one perpendicular to OA. | |

# Part 7 · The Crossnumber

*Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.*




## ACROSS

**1.** Radius in cm of a circle with a 24 cm chord 5 cm from the centre *(2 digits)*

**3.** Value of ∠C, in degrees, when ∠A = 75° in a cyclic quadrilateral *(3 digits)*

**4.** Distance in cm between two 7 cm chords whose radius works out to this value *(2 digits)*

## DOWN

**2.** A complete angle at the centre, in degrees *(3 digits)*

**3.** Sum of opposite angles of a cyclic quadrilateral, in degrees *(3 digits)*

**4.** Length in cm of a chord 6 cm from the centre of a 10 cm-radius circle *(2 digits)*

*Bonus questions that did not fit the grid — work them out anyway:* Length of a chord 5 cm from the centre of a 13 cm-radius circle; Half of 24 — or the length of a chord subtending 60° in a 12 cm-radius circle; Angle in a semicircle, in degrees; Angle subtended at the circle by an 80° arc

# Part 8 · Number Search & Magic Square

*Work out each answer, then find it hidden in the digit grid — across, down, diagonally or backwards.*

| 2 | 1 | 6 | 4 | 0 | 8 | 1 | 2 | 7 | 2 | 4 | 8 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 7 | 2 | 1 | 0 | 5 | 5 | 9 | 9 | 6 | 1 | 4 |
| 7 | 7 | 5 | 0 | 5 | 2 | 4 | 2 | 7 | 1 | 5 | 8 | 0 |
| 8 | 3 | 2 | 3 | 7 | 8 | 1 | 1 | 5 | 5 | 3 | 0 | 6 |
| 5 | 1 | 4 | 3 | 4 | 8 | 1 | 2 | 4 | 7 | 7 | 4 | 8 |
| 1 | 9 | 0 | 4 | 6 | 3 | 7 | 8 | 8 | 1 | 0 | 0 | 9 |
| 0 | 5 | 2 | 3 | 4 | 0 | 9 | 1 | 0 | 8 | 6 | 8 | 6 |
| 7 | 0 | 2 | 4 | 5 | 0 | 9 | 3 | 3 | 6 | 3 | 1 | 5 |
| 6 | 1 | 5 | 0 | 2 | 1 | 1 | 4 | 3 | 2 | 7 | 9 | 7 |
| 1 | 3 | 9 | 5 | 4 | 0 | 7 | 2 | 4 | 8 | 7 | 7 | 3 |
| 0 | 4 | 6 | 4 | 3 | 8 | 2 | 1 | 6 | 5 | 9 | 0 | 0 |
| 3 | 1 | 7 | 9 | 9 | 9 | 1 | 3 | 6 | 6 | 9 | 0 | 5 |
| 4 | 0 | 2 | 1 | 6 | 1 | 5 | 3 | 0 | 2 | 8 | 0 | 2 |

| | | | | |
|---|---|---|---|---|
| ☐ 180 | ☐ 360 | ☐ 105 | ☐ 24 | ☐ 12 |
| ☐ 90 | ☐ 40 | ☐ 16 | ☐ 13 | ☐ 70 |
| ☐ 60 | | | | |

**Magic Square — every row, column and diagonal adds to the same total.**

| | | |
|---|---|---|
| | | **6** |
| **3** | | **7** |
| **4** | **9** | |

*Magic constant: 15. Fill the empty cells.*

# Part 9 · Spot the Mistake

*Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.*

### ❓ Spot the Mistake

**Finding the length of a chord 8 cm from the centre of a 17 cm-radius circle**

**Step 1.** Use length ¿$2\sqrt{r^2-d^2}$ with $r=17$, $d=8$.

**Step 2.** $r^2-d^2=17^2-8^2=289-64=225$.

**Step 3.** $\sqrt{225}=15$, so the chord length is 15 cm.

_____ The first wrong step is Step _____ . What should it say?

_____

### ❓ Spot the Mistake

**Finding ∠C in cyclic quadrilateral ABCD with ∠A = 75°**

**Step 1.** Opposite angles of a cyclic quadrilateral sum to 180°.

**Step 2.** ∠A and ∠C are opposite, so ∠A + ∠C = 180°.

**Step 3.** ∠C = 180° + 75° = 255°.

_____ The first wrong step is Step _____ . What should it say?

_____

### ❓ Spot the Mistake

**The angle subtended by an 80° arc at the circle**

**Step 1.** Theorem 9 says the central angle is double the angle at the circle.

**Step 2.** So the angle at the circle equals the central angle times 2.

**Step 3.** $80\,^\circ \times 2 = 160\,^\circ$.

_____ The first wrong step is Step _____ . What should it say?

_____

# Part 10 · Quick-Fire Round

## Match each statement to the theorem number it describes

*Draw a line from each item on the left to its match on the right.*

| # | Left | Right (shuffled) |
|---|---|---|
| 1 | Equal chords subtend equal angles at the centre | Theorem 1 |
| 2 | The perpendicular from the centre bisects the chord | Theorem 9 |
| 3 | The longer of two chords is closer to the centre | Theorem 11 |
| 4 | The central angle is double the angle at the circle | Theorem 2 |
| 5 | Opposite angles of a cyclic quadrilateral sum to 180° | Theorem 8 |
| 6 | A unique circle passes through three non-collinear points | Theorem 10 |
| 7 | Equal angles at two points on the same side make four points concyclic | Theorem 7 |
| 8 | Chords equidistant from the centre are equal in length | Theorem 5 |

## True or False Sprint

*Everyone stands. Teacher reads a statement. True → stay standing. False → sit down.*

| # | Statement | Your call |
|---|---|---|
| 1 | A diameter is the longest possible chord of a circle. | ☐ True ☐ False |
| 2 | Every chord of a circle passes through the centre. | ☐ True ☐ False |
| 3 | The angle subtended by a diameter at any point on the circle is 90°. | ☐ True ☐ False |
| 4 | Equal chords of a circle are equidistant from the centre. | ☐ True ☐ False |
| 5 | Of two unequal chords, the shorter one is closer to the centre. | ☐ True ☐ False |
| 6 | A unique circle passes through any three points, collinear or not. | ☐ True ☐ False |
| 7 | The circumcentre of an obtuse-angled triangle lies outside the triangle. | ☐ True ☐ False |

| # | Statement | Your call |
|---|---|---|
| 8 | In a cyclic quadrilateral, opposite angles are always equal. | ☐ True ☐ False |
| 9 | The sum of opposite angles of a cyclic quadrilateral is 180°. | ☐ True ☐ False |
| 10 | Angles in the same segment of a circle, subtended by the same arc, are equal. | ☐ True ☐ False |
| 11 | A circle has only one line of reflection symmetry. | ☐ True ☐ False |
| 12 | The perpendicular from the centre to a chord bisects that chord. | ☐ True ☐ False |

## Two Truths and a Lie

*Two statements are true, one is false. Circle the lie — then correct it.*

### Set 1

(a) A circle has infinitely many lines of reflection symmetry.

(b) A circle has only rotational symmetry, not reflection symmetry.

(c) Every diameter of a circle is a line of reflection symmetry.

The lie is _____ . Corrected: _____

### Set 2

(a) The angle subtended by an arc at the centre is double the angle it subtends at a point on the circle outside the arc.

(b) Angles subtended by the same arc at different points of the same segment are equal.

(c) The angle subtended by a diameter at the centre is 90°.

The lie is _____ . Corrected: _____

### Set 3

(a) If two opposite angles of a quadrilateral sum to 180°, the quadrilateral is cyclic.

(b) Every quadrilateral can be inscribed in some circle.

(c) A rectangle can always be inscribed in a circle.

The lie is _____ . Corrected: _____

# Part 11 · The Escape Challenge

*Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.*

### 🔒 GATE 1 — The Chord Gate

A chord is 5 cm from the centre of a circle of radius 13 cm. Find half the chord length.

**Digit:** ☐

### 🔒 GATE 2 — The Arc Gate

An arc subtends 100° at the centre. Find the angle it subtends at a point on the circle outside the arc.

**Digit:** ☐

### 🔒 GATE 3 — The Semicircle Gate

AB is a diameter and C is on the circle. Find ∠ACB in degrees.

**Digit:** ☐

### 🔒 GATE 4 — The Cyclic Gate

In cyclic quadrilateral PQRS, ∠P = 65°. Find ∠R.

**Digit:** ☐

### 🔒 GATE 5 — The Theorem Gate

How many numbered theorems appear in the Learn First pages of this chapter (Theorem 1 through the last one stated)?

**Digit:** ☐

**EXIT CODE:** ☐ ☐ ☐ ☐ ☐

# Formula Sheet

*Cut this page out and keep it. Everything in this chapter, on one page.*

**Chord length from distance**

$$chord = 2\sqrt{r^2 - d^2}$$

**Angle at centre vs. at circle**

$$\angle(centre) = 2 \times \angle ¿$$

**Angle in a semicircle**

$$\angle ACB = 90^\circ \text{ when } AB \text{ is a diameter}$$

**Cyclic quadrilateral**

$$\angle A + \angle C = 180^\circ,\ \angle B + \angle D = 180^\circ$$

**Full angle at a point**

$$\sum \text{of angles at a point} = 360^\circ$$

# Full Solutions

*Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.*

## Part 3 · Guided Practice

**1.** r=10, d=6, $r^2-d^2=64$, $\sqrt{64}=8$, length=16 cm · **2.** double; 130°/2 = 65° · **3.** 180; 5x-10=180; 5x=190; x=38 · **4.** 180; half; 90

## Practice Set A

**1.** $24^2/2=12$ half-chord; $r=\sqrt{12^2+5^2}=\sqrt{144+25}=\sqrt{169}=13$ cm.

**2.** $\sqrt{17^2-8^2}=\sqrt{289-64}=\sqrt{225}=15$; chord ¿$2\times15=30$ cm.

**3.** $96\,^\circ/2=48\,^\circ$.

**4.** ∠ACB ¿$90\,^\circ$ (angle in a semicircle). ∠ABC ¿$180\,^\circ-90\,^\circ-35\,^\circ=55\,^\circ$.

**5.** ∠C ¿$180\,^\circ-68\,^\circ=112\,^\circ$; ∠D ¿$180\,^\circ-112\,^\circ=68\,^\circ$.

**6.** They are equidistant from the centre (Theorem 6).

**7.** AB, the longer chord, is closer to the centre (Theorem 8).

**8.** A circle is the set of all points in a plane equidistant from a fixed point, the centre; that fixed distance is the radius.

**9.** The circumcentre.

**10.** ∠R ¿$180\,^\circ-95\,^\circ=85\,^\circ$.

## Practice Set B

**1.** Half-chord ¿8 cm; $d=\sqrt{10^2-8^2}=\sqrt{100-64}=\sqrt{36}=6$ cm.

**2.** Distance of the 6 cm chord: $\sqrt{5^2-3^2}=4$; of the 8 cm chord: $\sqrt{5^2-4^2}=3$. On opposite sides, total distance ¿$4+3=7$ cm.

**3.** Triangle OAB has OA = OB = 12 (radii) and ∠AOB = 60°, so it is equilateral: AB ¿12 cm.

**4.** ABCD is cyclic so ∠ADC + ∠ABC = 180°. ∠ADE and ∠ADC are supplementary (E is on the extension of CD), so ∠ADE = 180° − ∠ADC = ∠ABC.

**5.** Distances: $d_1=\sqrt{r^2-25}$, $d_2=\sqrt{r^2-144}$, same side so $d_1-d_2=7$. Solving: $\sqrt{r^2-25}-\sqrt{r^2-144}=7$ gives $r=13$ cm.

**6.** No — opposite pairs must each sum to 180°. Here 80+100=180 ✓ but 110+70=180 ✓ as well, so it actually **is** possible (check both pairs, not just one).

**7.** No. The formula chord ¿$2\sqrt{r^2-d^2}$ is not linear in $d$, so doubling $d$ does not halve the chord in the simple way "CD = 2·AB" would need — a counter-example with numbers confirms this quickly.

**8.** MN is a diameter, so ∠MON — wait, more precisely ∠MPN = 90° for any P on the circle; here ∠MOP is the angle at O (not necessarily 90°) while ∠MNP, standing on diameter MN from point O side, need not be 90° either unless O is the vertex opposite

the diameter — the key fact is ∠MPN = 90° for any point P on the circle other than M, N.

## Practice Set C

**1.** The perpendicular bisector of a chord AB is the locus of points equidistant from A and B; since the centre O satisfies OA = OB, O lies on this perpendicular bisector.

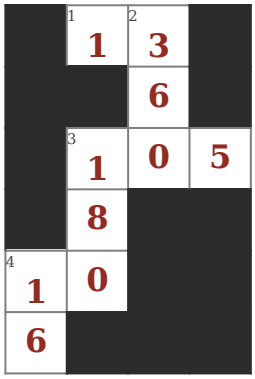
**2.** Where the chords cross, the two triangles formed use vertical angles and the equal chord lengths; congruence (SAS, from equal radii and equal chords) forces matching segments.

**3.** Each side of the hexagon subtends 60° at the centre, so the triangle formed is equilateral: side length ¿ $r$. Distance from centre to each side ¿ $r\cos 30\,°=\frac{\sqrt{3}}{2}r$.

**4.** A cyclic parallelogram has opposite angles equal (parallelogram property) and opposite angles summing to 180° (cyclic property); together these force every angle to be 90°, i.e. a rectangle.

**5.** By the chord-distance relationship, the chord perpendicular to OA has the smallest half-chord for a fixed distance $OA$, since any other chord through A has a larger perpendicular distance component along OA, making it longer, not shorter — the perpendicular chord uses the full available "shortening" effect of $d=OA$.

## Part 7 · Crossnumber — Solved Grid




**Across:** 1 = 13 · 3 = 105 · 4 = 10
**Down:** 2 = 360 · 3 = 180 · 4 = 16

## Part 8 · Number Search — Solved Grid

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 6 | 4 | 0 | 8 | 1 | 2 | 7 | 2 | 4 | 8 | 0 |
| 2 | 0 | 7 | 2 | **1** | **0** | **5** | 5 | 9 | 9 | 6 | 1 | 4 |
| 7 | 7 | 5 | 0 | 5 | 2 | 4 | 2 | 7 | **1** | 5 | 8 | 0 |
| 8 | 3 | 2 | 3 | 7 | 8 | 1 | **1** | 5 | 5 | **3** | 0 | 6 |
| 5 | 1 | 4 | **3** | 4 | 8 | 1 | **2** | 4 | 7 | 7 | 4 | 8 |
| 1 | 9 | 0 | 4 | **6** | 3 | 7 | 8 | 8 | 1 | 0 | **0** | 9 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 5 | **2** | 3 | 4 | **0** | 9 | 1 | 0 | 8 | 6 | **8** | 6 |
| 7 | 0 | 2 | **4** | 5 | 0 | 9 | 3 | 3 | 6 | 3 | **1** | 5 |
| 6 | 1 | 5 | 0 | 2 | 1 | 1 | 4 | 3 | 2 | 7 | 9 | 7 |
| 1 | 3 | 9 | 5 | 4 | 0 | 7 | 2 | 4 | 8 | **7** | 7 | 3 |
| **0** | **4** | 6 | 4 | 3 | 8 | 2 | **1** | **6** | 5 | 9 | **0** | 0 |
| 3 | 1 | 7 | 9 | 9 | 9 | 1 | 3 | **6** | 6 | **9** | **0** | 5 |
| 4 | 0 | 2 | 1 | 6 | 1 | 5 | 3 | **0** | 2 | 8 | 0 | 2 |

## Part 8 · Magic Square — Solved (constant 15)

| | | |
|---|---|---|
| **8** | **1** | **6** |
| **3** | **5** | **7** |
| **4** | **9** | **2** |

## Part 9 · Spot the Mistake

| Problem | First wrong step | It should say |
|---|---|---|
| Finding the length of a chord 8 cm from the centre of a 17 cm-radius circle | Step 1 | The formula already has the factor of 2 built in: length ¿$2\sqrt{225}=2\times15=30$ cm — forgetting the 2 is the most common slip with this formula. |
| Finding ∠C in cyclic quadrilateral ABCD with ∠A = 75° | Step 1 | It is 180° **minus** 75°, not plus: ∠C = 180° − 75° = 105°. An angle inside a quadrilateral can never exceed 180°, which should have been a warning sign. |
| The angle subtended by an 80° arc at the circle | Step 1 | If the central angle is double the circle angle, the circle angle is **half** the central angle, not double it: $\frac{80^\circ}{2}=40^\circ$. |

## Part 10 · Matching

| # | Left | Correct match |
|---|---|---|
| 1 | Equal chords subtend equal angles at the centre | Theorem 2 |
| 2 | The perpendicular from the centre bisects the chord | Theorem 5 |
| 3 | The longer of two chords is closer to the centre | Theorem 8 |
| 4 | The central angle is double the angle at the circle | Theorem 9 |
| 5 | Opposite angles of a cyclic quadrilateral sum to 180° | Theorem 11 |
| 6 | A unique circle passes through three non-collinear points | Theorem 1 |
| 7 | Equal angles at two points on the same side make four points concyclic | Theorem 10 |
| 8 | Chords equidistant from the centre are equal in length | Theorem 7 |

## Part 10 · True or False

1-T · 2-F · 3-T · 4-T · 5-F · 6-F · 7-T · 8-F · 9-T · 10-T · 11-F · 12-T

## Part 10 · Two Truths and a Lie

| Set | The lie | Correction |
|---|---|---|
| 1 | b | A circle actually has both — every diameter is a line of reflection symmetry, so statement (b) is the false one. |
| 2 | c | A diameter subtends a straight angle, 180°, at the centre — not 90°. The 90° result is what a diameter subtends at a point ON the circle, not at the centre. |
| 3 | b | Only cyclic quadrilaterals (where opposite angles sum to 180°) can be inscribed in a circle — a general quadrilateral cannot. |

## Part 11 · The Escape Challenge

### EXIT CODE: 12 · 50 · 90 · 115 · 12

(Answer: 12)  (Answer: 50)  (Answer: 90)  (Answer: 115)  (Answer: 12)

*Every result in this chapter grew out of one sentence: all points on a circle are the same distance from the centre. Keep coming back to that sentence — it answers more questions than it looks like it should. ○*